Part restrictions: rational grading in Description Logics

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9th Panhellenic Logic Symposium
July 15, 2013, Athens

[1] Supported by the Contract 42/2013 with SF of Sofia University

Outline

Description Logics

Description Logics are logical formalism widely used in knowledge representation systems.

- explicit knowledge representation in form of taxonomy
- inferring new knowledge out of the presented structure by means of a specialized inference engine

Representation language

Concept language—comprises expressions with only unary and binary predicates.

- *concepts*: Person, Female
  interpreted as subsets of the model
- *roles*: $\mathrm{hasChild}$
  interpreted as binary relations in the model
- *constructors*: $\sqcap$, $\sqcup$, $\neg$, $\forall R$, $\exists R$

$$\mathrm{Person} \sqcap \neg\mathrm{Female}$$

$$\exists\mathrm{hasChild}.\mathrm{Male}$$

$$\geqslant 3\ \mathrm{hasChild}$$

Concept languages differ mainly in the constructors adopted for building complex concepts and roles.

They are compared with respect to:

- their expressiveness, and
- the complexity of inferences in them

Concept language $\mathcal{AL}$, syntax

$C,\ D$	$\to$	$A \mid$	(atomic concept)
		$\top \mid$	(universal concept)
		$\bot \mid$	(empty concept)
		$\neg A \mid$	(atomic negation)
		$C \sqcap D \mid$	(intersection)
		$\forall P.C \mid$	(universal role quantification)
		$\exists P.\top$	(restricted existential role quantification)

Concept language $\mathcal{AL}$, semantics

Interpretation $\mathcal{I} = (\triangle^{\mathcal{I}}, \cdot^{\mathcal{I}})$:

$$
\begin{array}{rcl}
A^{\mathcal{I}} & \subseteq & \triangle^{\mathcal{I}} \\
P^{\mathcal{I}} & \subseteq & \triangle^{\mathcal{I}} \times \triangle^{\mathcal{I}} \\
\top^{\mathcal{I}} & = & \triangle^{\mathcal{I}} \\
\bot^{\mathcal{I}} & = & \varnothing \\
(\neg A)^{\mathcal{I}} & = & \triangle^{\mathcal{I}} \setminus A^{\mathcal{I}} \\
(C \sqcap D)^{\mathcal{I}} & = & C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(\forall P.C)^{\mathcal{I}} & = & \{a \in \triangle^{\mathcal{I}} \mid \forall b : (a, b) \in P^{\mathcal{I}} \rightarrow b \in C^{\mathcal{I}}\} \\
(\exists P.\top)^{\mathcal{I}} & = & \{a \in \triangle^{\mathcal{I}} \mid \exists b : (a, b) \in P^{\mathcal{I}}\}
\end{array}
$$

Main inference tasks

Satisfiability. A concept C is satisfiable, if there exists an interpretation $\mathcal{I}$, such that $C^{\mathcal{I}} \neq \varnothing$.

Subsumption. $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ for any interpretation $\mathcal{I}$.

Proposition. $C \sqsubseteq D$ iff $C \sqcap \neg D$ is unsatisfiable.

The complexity of a Description Logic (concept language) is the complexity of checking the subsumption between its concepts.

Extensions of $\mathcal{AL}$

- $C \sqcup D \quad (\mathcal{U})$
  Union of concepts
  $$(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$$
- $\exists R.C \quad (\mathcal{E})$
  full Existential role quantification
  $$(\exists R.C)^{\mathcal{I}} = \{a \in \triangle^{\mathcal{I}} \mid \exists b : (a, b) \in R^{\mathcal{I}} \;\&\; b \in C^{\mathcal{I}}\}$$
- $\neg C \quad (\mathcal{C})$
  Complement (of non-atomic concepts)
  $$(\neg C)^{\mathcal{I}} = \triangle^{\mathcal{I}} \setminus C^{\mathcal{I}}$$

Counting constructors

- $\geqslant nR$ and $\leqslant nR$, $n \geq 0$ $(\mathcal{N})$
  Number restrictions

$$(\geqslant nR)^{\mathcal{I}} = \{a \in \triangle^{\mathcal{I}} \mid card\{b : (a,b) \in R^{\mathcal{I}}\} \geqslant n\}$$

- $\geqslant nR.C$ and $\leqslant nR.C$, $n \geq 0$ $(\mathcal{Q})$
  Qualified number restrictions

$$(\geqslant nR.C)^{\mathcal{I}} = \{a \in \triangle^{\mathcal{I}} \mid card\{b : (a,b) \in R^{\mathcal{I}} \;\&\; b \in C^{\mathcal{I}}\} \geqslant n\}$$

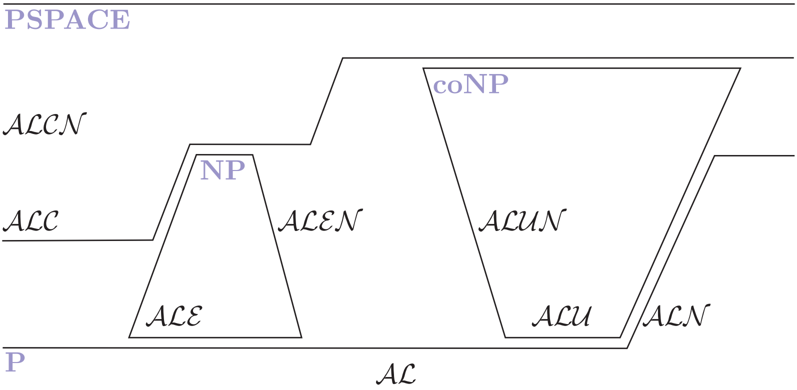
Extensions of $\mathcal{AL}$

Extensions of $\mathcal{AL}$: $\mathcal{AL}[\mathcal{U}][\mathcal{E}][\mathcal{C}][\mathcal{N}/\mathcal{Q}]$

Sources of complexity

- AND-complexity
  $\exists R.C$: NP-hardness
- OR-complexity
  $C \sqcup D$: co-NP-hardness

Complexity of $\mathcal{AL}$-languages



Constructors for part of the whole

- $MrR.C$ and $WrR.C$ $(\mathcal{P})$
  Part restrictions
- $MrR.A^{\varepsilon}$ and $WrR.A^{\varepsilon}$ $(\mathcal{P}^{\varepsilon})$
  limited part restrictions

$r \in \mathbb{Q} \cap (0, 1)$,
R is a role, C is a concept, A^{ε} is a (negated) atomic concept

$MrR.C$: *M*ore than r-part of all R-successors of the current object has the property C.

$$(MrR.C)^{\mathcal{I}} = \left\{ a \in \triangle^{\mathcal{I}} | \ card\{R^{\mathcal{I}}(a, C^{\mathcal{I}})\} > r.card\{R^{\mathcal{I}}(a)\} \right\}$$

Examples of concepts with part restrictions

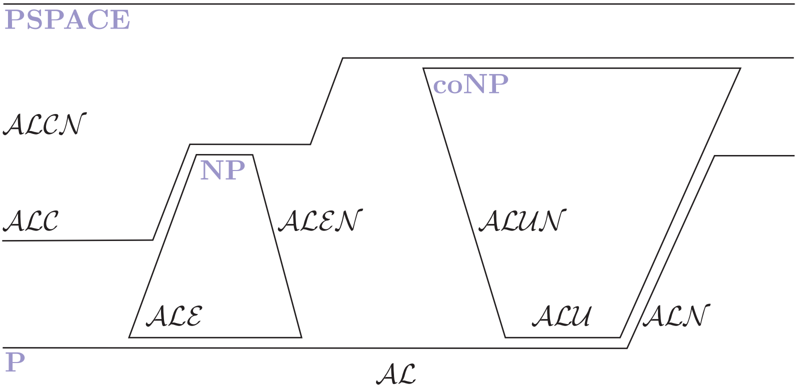
$$M_{1\backslash 2}vote.Yes$$

$$M_{2\backslash 3}vote.Yes$$

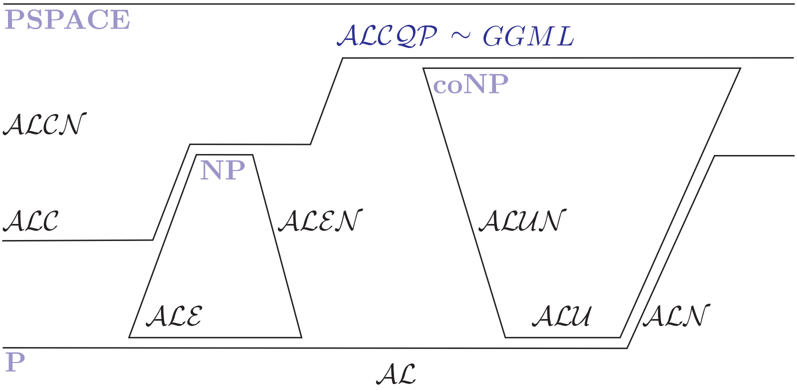
Extensions of $\mathcal{AL}$ with part restrictions

$$\mathcal{AL}[\mathcal{U}][\mathcal{E}][\mathcal{C}][\mathcal{N}/\mathcal{Q}]\mathcal{P}/\mathcal{P}^{\varepsilon}$$

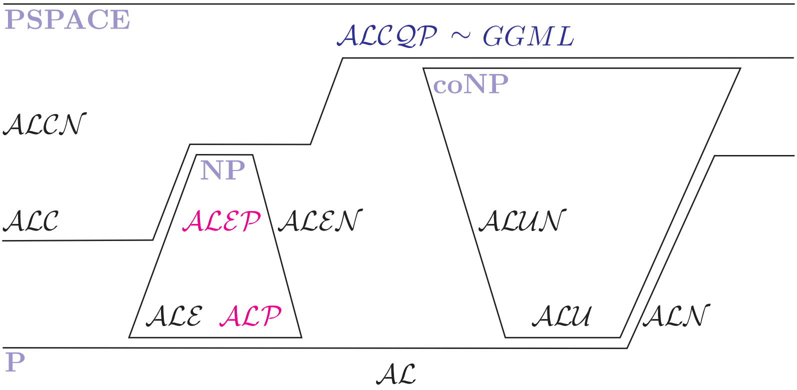
Complexity of $\mathcal{AL}$-languages, a recall



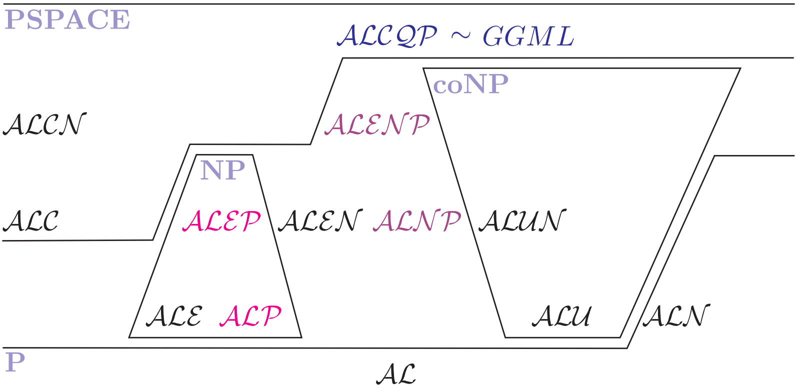
Complexity of $\mathcal{AL}$-languages with part restrictions (1)



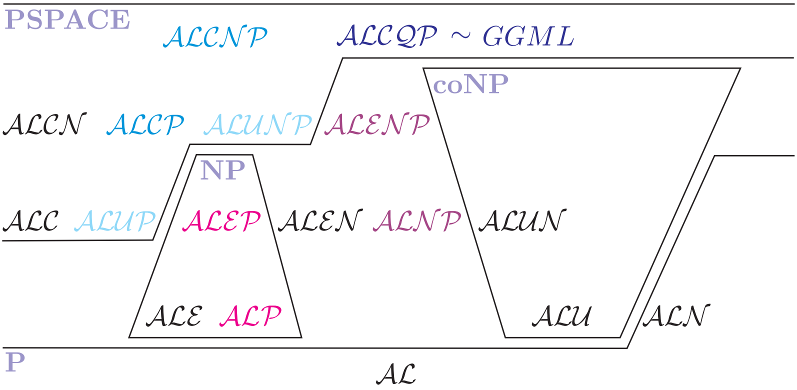
Complexity of $\mathcal{AL}$-languages with part restrictions (2)



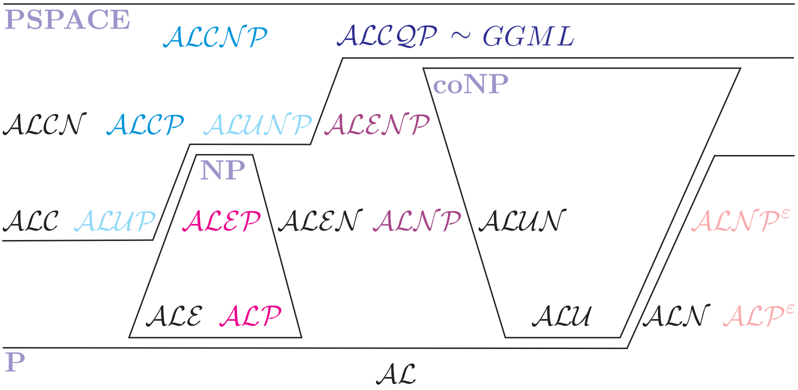
Complexity of $\mathcal{AL}$-languages with part restrictions (3)



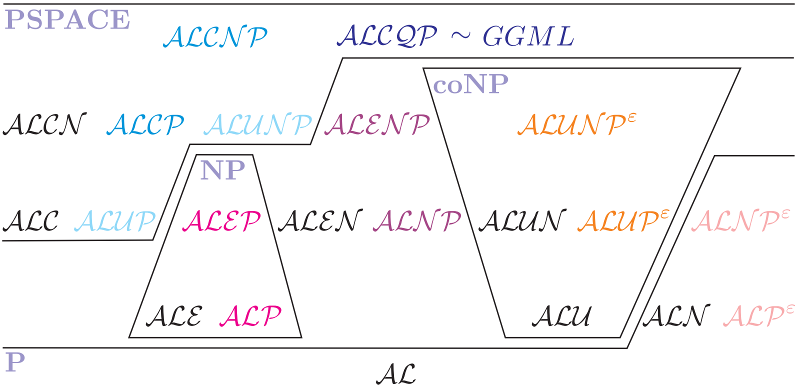
Complexity of $\mathcal{AL}$-languages with part restrictions (4)



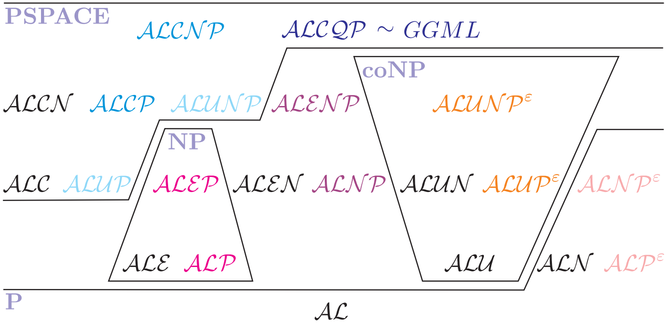
$\mathcal{AL}$-languages with limited part restrictions (1)



$\mathcal{AL}$-languages with limited part restrictions (2)



What comes next?



$$? \ \mathcal{P}/\mathcal{P}^{\varepsilon} \ \leftrightarrow \ \mathcal{R}$$

Thank you for your attention!

Statements

Theorem. Subsumption and satisfiability in $\mathcal{ALP}^{\varepsilon}$ can be decided in polynomial time, independently of the notation for numbers.

Theorem. Subsumption and satisfiability in $\mathcal{ALNP}^{\varepsilon}$ can be decided in polynomial time, in case the numbers in the concepts are represented in unary notation.

cs: Dealing with contradictions (1)

Clashes

Propositional contradictions

CL1. $\{x : \bot\}$

CL2. $\{x : A, x : \neg A\}$

Number contradictions (in the language with $\mathcal{N}$)

CL3. $\{x : (\leq nR)\} \cup \{xRy_1, ..., xRy_{n+1}\}$

cs: Dealing with contradictions (2)

Clusters with part restrictions $Cl_x(R, C)$

$$\{x : Mr_1 R.C,\ x : Mr_2 R.\neg C,\ x : Wr_3 R.C,\ x : Wr_4 R.\neg C\}$$

Cluster consistency, necessary and sufficient conditions

- $r_1 + r_2 < 1$
- $r_1 < r_4$
- $r_2 < r_3$
- $r_3 + r_4 \geq 1$

Rational contradictions—cluster inconsistency

cs: Reducing number of constraints (1)

Reducing number of variables:
two kinds of variables—active ones and dead ends.

$$x : (\geq nR),\ n > 1\ \&\ Cl_x(R, A^{\varepsilon})$$

$$x : Mr_1 R.A,\ x : Mr_2 R.\neg A$$

$\mathcal{V} = \mathcal{V}_{act} \cup \mathcal{V}_{d.e.},\ \mathcal{V}_{act} \cap \mathcal{V}_{d.e.} = \varnothing$
$\mathcal{V}_{act} = \{\check{x}, \check{y}, ...\}$ - "active" variables
$\mathcal{V}_{d.e.} = \{\bar{x}, \bar{y}, ...\}$ - "dead ends"

cs: Reducing number of constraints (2)

$\rightarrow_{M_{act}}$-rule: $S \rightarrow_{M_{act}} S \cup \{xRy,\ y : A^{\varepsilon}\}$;

If $x \in \mathcal{V}_{act}$, $x : MrR.A^{\varepsilon} \in S$,
$\nexists z(xRz \in S)$, and
$y \in \mathcal{V}_{act}$ is new

$\rightarrow_{M_{d.e.}}$-rule: $S \rightarrow_{M_{d.e.}} S \cup \{xRy,\ y : A^{\varepsilon}\}$;

If $x \in \mathcal{V}_{act}$, $x : MrR.A^{\varepsilon} \in S$,
$\exists z(xRz \in S)$, $|R(x, A^{\varepsilon})| \leq r|R(x)|$, and
$y \in \mathcal{V}_{d.e.}$ is new

+ sophisticated strategy for the rules' application

cs: Reducing number of constraints (3)