Decidability of modal logics for dynamic contact relations

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Outline

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Mereotopology

Dynamic mereotopological relations

The modal logic

The first reduct

The second reduct

Open problems

Outline

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Mereotopology

Mereotopology = Mereology + topological relations.

- mereology is an ontological discipline, theory of “Parts and Wholes”(see [9]);
- main relations in mereology: *part-of*, *overlap* and *underlap*;
- its mathematical equivalent are complete Boolean algebras without the zero element (Tarski, see [9]);
- the mathematical equivalent to mereotopology Contact algebras ([1], [2], [10]).

Contact algebras = Boolean algebras+contact-based relations.

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Contact algebras - general definition

Definition (Contact algebra)

$(\underline{B}, \mathrm{C}) = (B, 0, 1, ., +, *, \mathrm{C})$ is called a *contact algebra* if $\underline{B}$ is a Boolean algebra and C is a binary relation satisfying:

$$x \,\mathrm{C}\, y \Longrightarrow x \neq 0 \,\&\, y \neq 0, \qquad x \,\mathrm{C}\, y \Longrightarrow y \,\mathrm{C}\, x,$$
$$x \,\mathrm{C}\, (y+z) \Longleftrightarrow x \,\mathrm{C}\, y \text{ or } x \,\mathrm{C}\, z, \qquad x.y \neq 0 \Longrightarrow x \,\mathrm{C}\, y.$$

Lemma

Let $\mathbb{X}$ be a topological space. Then $(\underline{B}, \mathrm{C})$ is a contact algebra, where $\underline{B}$ is the Boolean algebra of the regular closed sets of $\mathbb{X}$ and C is the topological contact in $\mathbb{X}$.

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Contact algebras - standard definition

Lemma

Let (X, R) *be a frame, where* R *is a reflexive and symmetric relation.* $(\underline{B}, \mathrm{C})$ *is a contact algebra, where* $\underline{B}$ *is the Boolean algebra of the subsets of* X *and* C *is defined for* $x, y \subseteq W$*:*

$$x \,\mathrm{C}\, y \quad \textit{iff} \quad \exists a \in x, \exists b \in y, a \,\mathrm{R}\, b.$$

Definition (Mereotopological structure)

Let $(B, 0, 1, ., +, *, \mathrm{C})$ be a contact algebra.
$\underline{W} = (W, \leq, \mathrm{O}, \mathrm{U}, \mathrm{C})$ a *(static) mereotopological structure* if $W \neq \emptyset$, $W \subseteq B$ and $\leq$, O and U are defined

$$x \leq y \text{ iff } x.y* = 0,\ x \,\mathrm{O}\, y \text{ iff } x.y \neq 0,\ x \,\mathrm{U}\, y \text{ iff } x + y \neq 1.$$

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Stable and unstable relations - standard definition

Definition (Standard dynamic mereotopological structure)

Let $I \neq \emptyset$ and for every $i \in I$, $\underline{W}_i = (W_i, \leq_i, \mathrm{O}_i, \mathrm{U}_i, \mathrm{C}_i)$ be a static structure. Let $W \subseteq \prod_{i \in I} W_i$, $W \neq \emptyset$. Then for $x, y \in W$:

$x \leq y$	iff	$\forall i \in I, x_i \leq_i y_i$	*stable part-of*,
$x \mathrm{o} y$	iff	$\forall i \in I, x_i \mathrm{O}_i y_i$	*stable overlap*,
$x \mathrm{u} y$	iff	$\forall i \in I, x_i \mathrm{U}_i y_i$	*stable underlap*,
$x \mathrm{c} y$	iff	$\forall i \in I, x_i \mathrm{C}_i y_i$	*stable contact*,
$x \preceq y$	iff	$\exists i \in I, x_i \leq_i y_i$	*unstable part-of*,
$x \mathrm{O} y$	iff	$\exists i \in I, x_i \mathrm{O}_i y_i$	*unstable overlap*,
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Intuition

The intuition behind the formal definition is

- I are the moments of time;
- $\underline{W}_i$ are snapshots of the environment;
- $x \in W$ are histories of changing regions;
- *stable* means *always*;
- *unstable* means *sometimes*.

Integrated language of spacial and temporal primitives, which cannot be considered independantly (ideas of Whitehead [11] and de Laguna [4]).

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Stable and unstable relations - general definition

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$(W, \leq, \mathrm{o}, \mathrm{u}, \mathrm{c}, \preceq, \mathrm{O}, \mathrm{U}, \mathrm{C})$ is a *dynamic structure* if it satisfies:

(M1) $x \leq x$

(M2) $x \leq y \,\&\, y \leq z \Rightarrow x \leq z$

(M3) $x \leq y \,\&\, y \leq x \Rightarrow x = y$

(M4) $x \,\mathrm{O}\, y \Rightarrow y \,\mathrm{O}\, x$

(M5) $x \,\mathrm{O}\, y \Rightarrow x \,\mathrm{O}\, x$

(M6) $x \,\mathrm{O}\, y \,\&\, y \leq z \Rightarrow x \,\mathrm{O}\, z$

(M7) $x \,\mathrm{O}\, x$ or $x \leq y$

(M8) $x \,\mathrm{U}\, y \Rightarrow y \,\mathrm{U}\, x$

(M9) $x \,\mathrm{U}\, y \Rightarrow x \,\mathrm{U}\, x$

(M10) $x \leq y \,\&\, y \,\mathrm{U}\, z \Rightarrow x \,\mathrm{U}\, z$

(C1) $x \,\mathrm{C}\, y \Rightarrow y \,\mathrm{C}\, x$

(C2) $x \,\mathrm{O}\, y \Rightarrow x \,\mathrm{C}\, y$

(C3) $x \,\mathrm{C}\, y \Rightarrow x \,\mathrm{O}\, x$

(M11) $y \,\mathrm{U}\, y$ or $x \leq y$

(M12) $x \leq y$ or $x \,\mathrm{O}\, z$ or $y \,\mathrm{U}\, z$

(M13) $x \,\mathrm{O}\, x$ or $x \,\mathrm{U}\, x$

(M14) $x \preceq x$

(M15) $x \leq y \,\&\, y \preceq z \Rightarrow x \preceq z$

(M16) $x \preceq y \,\&\, y \leq z \Rightarrow x \preceq z$

(M17) $x \,\mathrm{o}\, y \Rightarrow y \,\mathrm{o}\, x$

(M18) $x \,\mathrm{o}\, y \Rightarrow x \,\mathrm{o}\, x$

(M19) $x \,\mathrm{o}\, y \,\&\, y \leq z \Rightarrow x \,\mathrm{o}\, z$

(M20) $x \,\mathrm{o}\, y \,\&\, y \preceq z \Rightarrow x \,\mathrm{O}\, z$

(C4) $x \,\mathrm{C}\, y \,\&\, y \leq z \Rightarrow x \,\mathrm{C}\, z$

(C5) $x \,\mathrm{c}\, y \Rightarrow y \,\mathrm{c}\, x$

(C6) $x \,\mathrm{o}\, y \Rightarrow x \,\mathrm{c}\, y$

(M21) $x \,\mathrm{o}\, x$ or $x \preceq y$

(M22) $x \,\mathrm{o}\, z$ or $y \,\mathrm{U}\, z$ or $x \preceq y$

(M23) $x \,\mathrm{u}\, y \Rightarrow y \,\mathrm{u}\, x$

(M24) $x \,\mathrm{u}\, y \Rightarrow x \,\mathrm{u}\, x$

(M25) $x \leq y \,\&\, y \,\mathrm{u}\, z \Rightarrow x \,\mathrm{u}\, z$

(M26) $x \preceq y \,\&\, y \,\mathrm{u}\, z \Rightarrow x \,\mathrm{U}\, z$

(M27) $x \,\mathrm{O}\, z$ or $y \,\mathrm{u}\, z$ or $x \preceq y$

(M28) $y \,\mathrm{u}\, y$ or $x \preceq y$

(M29) $x \,\mathrm{o}\, x$ or $x \,\mathrm{U}\, x$

(M30) $x \,\mathrm{O}\, x$ or $x \,\mathrm{u}\, x$

(C7) $x \,\mathrm{c}\, y \Rightarrow x \,\mathrm{o}\, x$

(C8) $x \,\mathrm{c}\, y \,\&\, y \leq z \Rightarrow x \,\mathrm{c}\, z$

(C9) $x \,\mathrm{c}\, y \,\&\, y \preceq z \Rightarrow x \,\mathrm{C}\, z$

(C10) $z \,\mathrm{c}\, t \,\&\, x \,\overline{\mathrm{u}}\, y \,\&\, z \,\overline{\mathrm{O}}\, y \,\&\, t \,\overline{\mathrm{O}}\, x \Rightarrow x \,\mathrm{C}\, y$

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(M7) $x \mathrm{O} x$ or $x \leq y$

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(C9) $x \mathrm{c} y \,\&\, y \preceq z \Rightarrow x \mathrm{C} z$

(C10) $z \mathrm{c} t \,\&\, x \bar{\mathrm{u}} y \,\&\, z \bar{\mathrm{O}} y \,\&\, t \bar{\mathrm{O}} x \Rightarrow x \mathrm{C} y$

The first-order logic

Two classes: the standard structures - Σ_{std}, and the general structures - Σ_{gen}. Results ([5], [6], [7]):

- $FOL(\Sigma_{std}) = FOL(\Sigma_{gen})$;
- $FOL(\Sigma_{std})$ is complete w.r.t. (M1)-(M30), (C1)-(C10);
- $FOL(\Sigma_{std})$ (or $FOL(\Sigma_{gen})$ respectively) is hereditary undecidable (see [3]);
- the quantifier-free fragment of this logic is complete;
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The logic, axioms, definability

The polymodal logic of Σ_{std} and the universal relation A.

(M1)-(M30), (C1)-(C10) are definable, but (M3). Replace it with:

(M3') $x \,\overline{\mathrm{O}}\, x$ and $y \leq x \Rightarrow x = y$

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Definition

$(W, \leq, \mathrm{o}, \mathrm{u}, \mathrm{c}, \preceq, \mathrm{O}, \mathrm{U}, \mathrm{C})$ is a *non-standard structure* if it satisfies (M1),(M2),(M3'),(M3"),(M3"'),(M4)-(M30), (C1)-(C10). These structures form Σ_{nonstd}.

Lemma (P-morphism lemma)

For every non-standard structure $\underline{W}$ there is a general structure $\underline{W'}$ and a p-morphism from $\underline{W}$ onto $\underline{W'}$.

Proved via a generalization of Segerberg's Bulldozer method.

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Definition

$(W, \leq, \text{o}, \text{u}, \text{c}, \preceq, \text{O}, \text{U}, \text{C})$ is a *non-standard structure* if it satisfies (M1),(M2),(M3'),(M3''),(M3'''),(M4)-(M30), (C1)-(C10). These structures form Σ_{nonstd}.

Lemma (P-morphism lemma)

For every non-standard structure $\underline{W}$ there is a general structure $\underline{W'}$ and a p-morphism from $\underline{W}$ onto $\underline{W'}$.

Proved via a generalization of Segerberg's Bulldozer method.

Completeness

Theorem (Completeness theorem)

The following propositions are equivalent for every formula α:

(1) α *is theorem of the logic;*

(2) α *is true in every non-standard structure from* Σ_{nonstd}*;*

(3) α *is true in every general structure from* Σ_{gen}*;*

(4) α *is true in every standard structure from* Σ_{std}*.*

See [5], [7], [8] for detailed proofs. Here is a sketch:
(1) $\longrightarrow$ (2): soundness;
(1) $\longleftarrow$ (2): generated cannonical models;
(2) $\longrightarrow$ (3): every general structure is non-standard;
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The first reduct

The logic with the unstable contact C, without the stable contact c. Models of the form $\underline{W} = (W, \leq, \mathrm{o}, \mathrm{u}, \preceq, \mathrm{O}, \mathrm{U}, \mathrm{C})$.

Filtration: start from reduct model $(\underline{W}, v)$ and set of formulae Γ.

1. Γ is closed under sub-formulae;
2. $\langle R \rangle \top \in \Gamma$ for each of the modalities o, u, O and U where $\top$ is an arbitrary fixed MLDM tautology;
3. if $[R]\alpha \in \Gamma$ for some modality R then $[R]\alpha \in \Gamma$ for all modalities of the logic.

We build the filtered finite model $(\underline{W}', v')$ in the standard way.

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We build the filtered finite model $(\underline{W}', v')$ in the standard way.

The filtration - relations $\leq$ and $\preceq$

Relation $\leq'$: $[x] \leq' [y]$ holds iff the following conditions are met

$v(x, \langle \mathrm{O} \rangle \top) = 1$ implies $v(y, \langle \mathrm{O} \rangle \top) = 1$,
$v(x, \langle \mathrm{o} \rangle \top) = 1$ implies $v(y, \langle \mathrm{o} \rangle \top) = 1$,
$v(x, [\leq]\alpha) = 1$ implies $v(y, [\leq]\alpha) = 1$,
$v(y, [\mathrm{O}]\alpha) = 1$ implies $v(x, [\mathrm{O}]\alpha) = 1$,
$v(x, [\preceq]\alpha) = 1$ implies $v(y, [\preceq]\alpha) = 1$,
$v(y, [\mathrm{o}]\alpha) = 1$ implies $v(x, [\mathrm{o}]\alpha) = 1$,
$v(y, [\mathrm{C}]\alpha) = 1$ implies $v(x, [\mathrm{C}]\alpha) = 1$,

$v(y, \langle \mathrm{U} \rangle \top) = 1$ implies $v(x, \langle \mathrm{U} \rangle \top) = 1$,
$v(y, \langle \mathrm{u} \rangle \top) = 1$ implies $v(x, \langle \mathrm{u} \rangle \top) = 1$,
$v(y, [\geq]\alpha) = 1$ implies $v(x, [\geq]\alpha) = 1$,
$v(x, [\mathrm{U}]\alpha) = 1$ implies $v(y, [\mathrm{U}]\alpha) = 1$,
$v(y, [\succeq]\alpha) = 1$ implies $v(x, [\succeq]\alpha) = 1$,
$v(x, [\mathrm{u}]\alpha) = 1$ implies $v(y, [\mathrm{u}]\alpha) = 1$,
$v(y, [\mathrm{c}]\alpha) = 1$ implies $v(x, [\mathrm{c}]\alpha) = 1$.

Relation $\preceq'$: $[x] \preceq' [y]$ iff

$v(x, \langle \mathrm{o} \rangle \top) = 1$ implies $v(y, \langle \mathrm{O} \rangle \top) = 1$,
$v(x, [\preceq]\alpha) = 1$ implies $v(y, [\leq]\alpha) = 1$,
$v(y, [\mathrm{O}]\alpha) = 1$ implies $v(x, [\mathrm{o}]\alpha) = 1$,
$v(y, [\mathrm{C}]\alpha) = 1$ implies $v(x, [\mathrm{c}]\alpha) = 1$.

$v(y, \langle \mathrm{u} \rangle \top) = 1$ implies $v(x, \langle \mathrm{U} \rangle \top) = 1$,
$v(y, [\succeq]\alpha) = 1$ implies $v(x, [\geq]\alpha) = 1$,
$v(x, [\mathrm{U}]\alpha) = 1$ implies $v(y, [\mathrm{u}]\alpha) = 1$,

The filtration - relations $\leq$ and $\preceq$

Relation $\leq'$: $[x] \leq' [y]$ holds iff the following conditions are met

$v(x, \langle O \rangle \top) = 1$ implies $v(y, \langle O \rangle \top) = 1$,
$v(x, \langle o \rangle \top) = 1$ implies $v(y, \langle o \rangle \top) = 1$,
$v(x, [\leq]\alpha) = 1$ implies $v(y, [\leq]\alpha) = 1$,
$v(y, [O]\alpha) = 1$ implies $v(x, [O]\alpha) = 1$,
$v(x, [\preceq]\alpha) = 1$ implies $v(y, [\preceq]\alpha) = 1$,
$v(y, [o]\alpha) = 1$ implies $v(x, [o]\alpha) = 1$,
$v(y, [C]\alpha) = 1$ implies $v(x, [C]\alpha) = 1$,
$v(y, \langle U \rangle \top) = 1$ implies $v(x, \langle U \rangle \top) = 1$,
$v(y, \langle u \rangle \top) = 1$ implies $v(x, \langle u \rangle \top) = 1$,
$v(y, [\geq]\alpha) = 1$ implies $v(x, [\geq]\alpha) = 1$,
$v(x, [U]\alpha) = 1$ implies $v(y, [U]\alpha) = 1$,
$v(y, [\succeq]\alpha) = 1$ implies $v(x, [\succeq]\alpha) = 1$,
$v(x, [u]\alpha) = 1$ implies $v(y, [u]\alpha) = 1$,
$v(y, [c]\alpha) = 1$ implies $v(x, [c]\alpha) = 1$.

Relation $\preceq'$: $[x] \preceq' [y]$ iff

$v(x, \langle o \rangle \top) = 1$ implies $v(y, \langle O \rangle \top) = 1$,
$v(x, [\preceq]\alpha) = 1$ implies $v(y, [\leq]\alpha) = 1$,
$v(y, [O]\alpha) = 1$ implies $v(x, [o]\alpha) = 1$,
$v(y, [C]\alpha) = 1$ implies $v(x, [c]\alpha) = 1$.
$v(y, \langle u \rangle \top) = 1$ implies $v(x, \langle U \rangle \top) = 1$,
$v(y, [\succeq]\alpha) = 1$ implies $v(x, [\geq]\alpha) = 1$,
$v(x, [U]\alpha) = 1$ implies $v(y, [u]\alpha) = 1$,

The filtration - relations o, O, u, U and C

Relation o′: $[x]\ \mathrm{o}'\ [y]$ iff

$v(x, \langle o \rangle \top) = 1$ $\quad$ $v(y, \langle o \rangle \top) = 1$

$v(x, [o]\alpha) = 1$ implies $v(y, [\leq]\alpha) = 1$, $\quad$ $v(y, [o]\alpha) = 1$ implies $v(x, [\leq]\alpha) = 1$,

$v(x, [O]\alpha) = 1$ implies $v(y, [\preceq]\alpha) = 1$, $\quad$ $v(y, [O]\alpha) = 1$ implies $v(x, [\preceq]\alpha) = 1$.

Relation O′: $[x]\ \mathrm{O}'\ [y]$ iff

$v(x, \langle O \rangle \top) = 1$ $\quad$ $v(y, \langle O \rangle \top) = 1$

$v(x, [O]\alpha) = 1$ implies $v(y, [\leq]\alpha) = 1)$, $\quad$ $v(y, [O]\alpha) = 1$ implies $v(x, [\leq]\alpha) = 1)$.

Relation u′: $[x]\ \mathrm{u}'\ [y]$ iff

$v(x, \langle u \rangle \top) = 1$ $\quad$ $v(y, \langle u \rangle \top) = 1$

$v(x, [u]\alpha) = 1$ implies $v(y, [\geq]\alpha) = 1$, $\quad$ $v(y, [u]\alpha) = 1$ implies $v(x, [\geq]\alpha) = 1$,

$v(x, [U]\alpha) = 1$ implies $v(y, [\succeq]\alpha) = 1$, $\quad$ $v(y, [U]\alpha) = 1$ implies $v(x, [\succeq]\alpha) = 1$.

Relation U′: $[x]\ \mathrm{U}'\ [y]$ iff

$v(x, \langle U \rangle \top) = 1$ $\quad$ $v(y, \langle U \rangle \top) = 1$

$v(x, [U]\alpha) = 1$ implies $v(y, [\geq]\alpha) = 1$, $\quad$ $v(y, [U]\alpha) = 1$ implies $v(x, [\geq]\alpha) = 1$.

Relation C′: $[x]\ \mathrm{C}'\ [y]$ iff

$v(x, \langle O \rangle \top) = 1$ $\quad$ $v(y, \langle O \rangle \top) = 1$

$v(x, [C]\alpha) = 1$ implies $v(y, [\leq]\alpha) = 1)$, $\quad$ $v(y, [C]\alpha) = 1$ implies $v(x, [\leq]\alpha) = 1)$.

The second reduct

The logic with both contacts, without the unstable part-of $\preceq$.
Models of the form $\underline{W} = (W, \leq, \mathrm{o}, \mathrm{u}, \mathrm{c}, \mathrm{O}, \mathrm{U}, \mathrm{C})$.

Filtration: start from reduct model $(\underline{W}, v)$ and Γ.

1. Γ is closed under sub-formulae;
2. if $\alpha \in \Gamma$ and α does not start with $[\leq]$ (i.e. α is not in the form of $[\leq]\beta$) then $[\leq]\alpha \in \Gamma$ and $[\leq]\neg\alpha \in \Gamma$.

The filtered finite model $(\underline{W}', v')$ is build standardly.

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The filtration

Relation $\leq'$ is defined standardly for S4 modality:

$$\text{for all } [\leq]\alpha \in \Gamma,\ v(x, [\leq]\alpha) = 1 \text{ implies } v(y, [\leq]\alpha) = 1.$$

$[x]\ O'\ [y] \leftrightarrow \exists z, t \in W, [z] \leq' [x], [t] \leq' [y]$ and $z\ O\ t$.
$[x]\ U'\ [y] \leftrightarrow \exists z, t \in W, [x] \leq' [z], [y] \leq' [t]$ and $z\ U\ t$.
$[x]\ o'\ [y] \leftrightarrow \exists z, t \in W, [z] \leq' [x], [t] \leq' [y]$ and $z\ o\ t$.
$[x]\ u'\ [y] \leftrightarrow \exists z, t \in W, [x] \leq' [z], [y] \leq' [t]$ and $z\ u\ t$.
$[x]\ c'\ [y] \leftrightarrow \exists z, t \in W, [z] \leq' [x], [t] \leq' [y]$ and $z\ c\ t$.
$[x]\ C'\ [y] \leftrightarrow \exists z, t \in W, [z] \leq' [x], [t] \leq' [y]$ and $z\ C\ t$.

The filtration

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$[x]\ \mathrm{U}'\ [y] \leftrightarrow \exists z, t \in W,\ [x] \leq' [z],\ [y] \leq' [t]$ and $z\ \mathrm{U}\ t$.
$[x]\ \mathrm{o}'\ [y] \leftrightarrow \exists z, t \in W,\ [z] \leq' [x],\ [t] \leq' [y]$ and $z\ \mathrm{o}\ t$.
$[x]\ \mathrm{u}'\ [y] \leftrightarrow \exists z, t \in W,\ [x] \leq' [z],\ [y] \leq' [t]$ and $z\ \mathrm{u}\ t$.
$[x]\ \mathrm{c}'\ [y] \leftrightarrow \exists z, t \in W,\ [z] \leq' [x],\ [t] \leq' [y]$ and $z\ \mathrm{c}\ t$.
$[x]\ \mathrm{C}'\ [y] \leftrightarrow \exists z, t \in W,\ [z] \leq' [x],\ [t] \leq' [y]$ and $z\ \mathrm{C}\ t$.

Open problems

Open problems and further developement of the dynamic mereotopological relations and logics for them:

- the decidability of the full modal logic;
- the complexity of the modal logic (if it is decidable);
- the complexity of the decidable reducts;
- addition of more mereotopological relations;
- addition of more temporal constructions - e.g. *before*, *until*, *since*, etc.

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Thank you!!

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