PLS 2013

The road from Leibniz to Turing

From Syllogisms to Computations

(Συλλογισμοί) (Υπολογισμοί)

Tribute to Alan Turing

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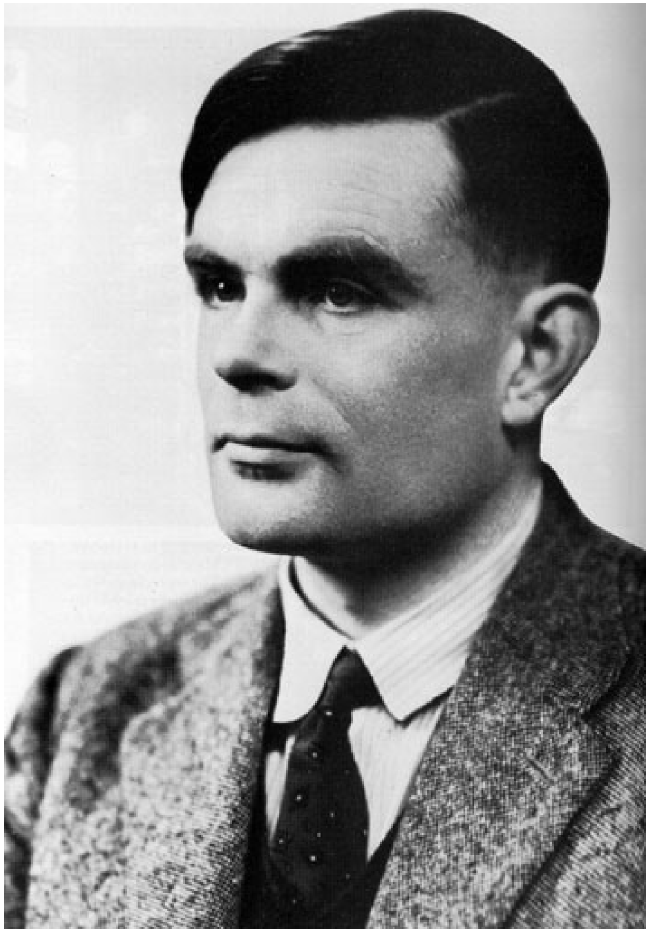
Engines of Logic

- Our life today is unimaginable without computers. Rapid development during the last seventy years. Our children will be living in a new electronic information world.
- But what has led to the invention and evolution of computers and Computer Science?
- Development was based more on the deep tradition of Mathematical Logic rather than the various technological innovations.
- Some names related to this journey from syllogisms to computations:
- Aristotle, Leibniz, Boole, Frege, Russel, Hilbert, Gödel, Turing and Von Neumann.
- More than anyone else, Turing contributed to the evolution of the contemporary world.

The Journey

- Leibniz was dreaming of reducing all human syllogisms to computations and constructing powerful engines that would execute such computations.
- Frege had created a system of rules that could offer a reasonable explanation for all human deductive arguments.
- In 1930, in his doctoral dissertation, Gödel had proven that Frege's rules were complete, thus answering the question raised by Hilbert two years earlier. Hilbert had also tried to find a computational procedure by which it would always be possible, given certain premises and a proposed conclusion (written in the symbolism which is now known as First Order Logic) to decide whether this conclusion can be deduced from these premises by using the given rules. The problem of finding such a procedure is known as Hilbert's Entscheidungsproblem (literally "decision problem"). Hilbert was looking for an algorithm of unprecedented range. Basically, an algorithm for the Entscheidungsproblem would have reduced all human deductive thought to mere computations. To a considerable extent, this would be a realization of Leibniz's dream.
- Alan Turing began to search for a way to prove that such an algorithm does not exist. It took a strict definition of the concept "algorithm" in order to prove this nonexistence.

Alan Turing



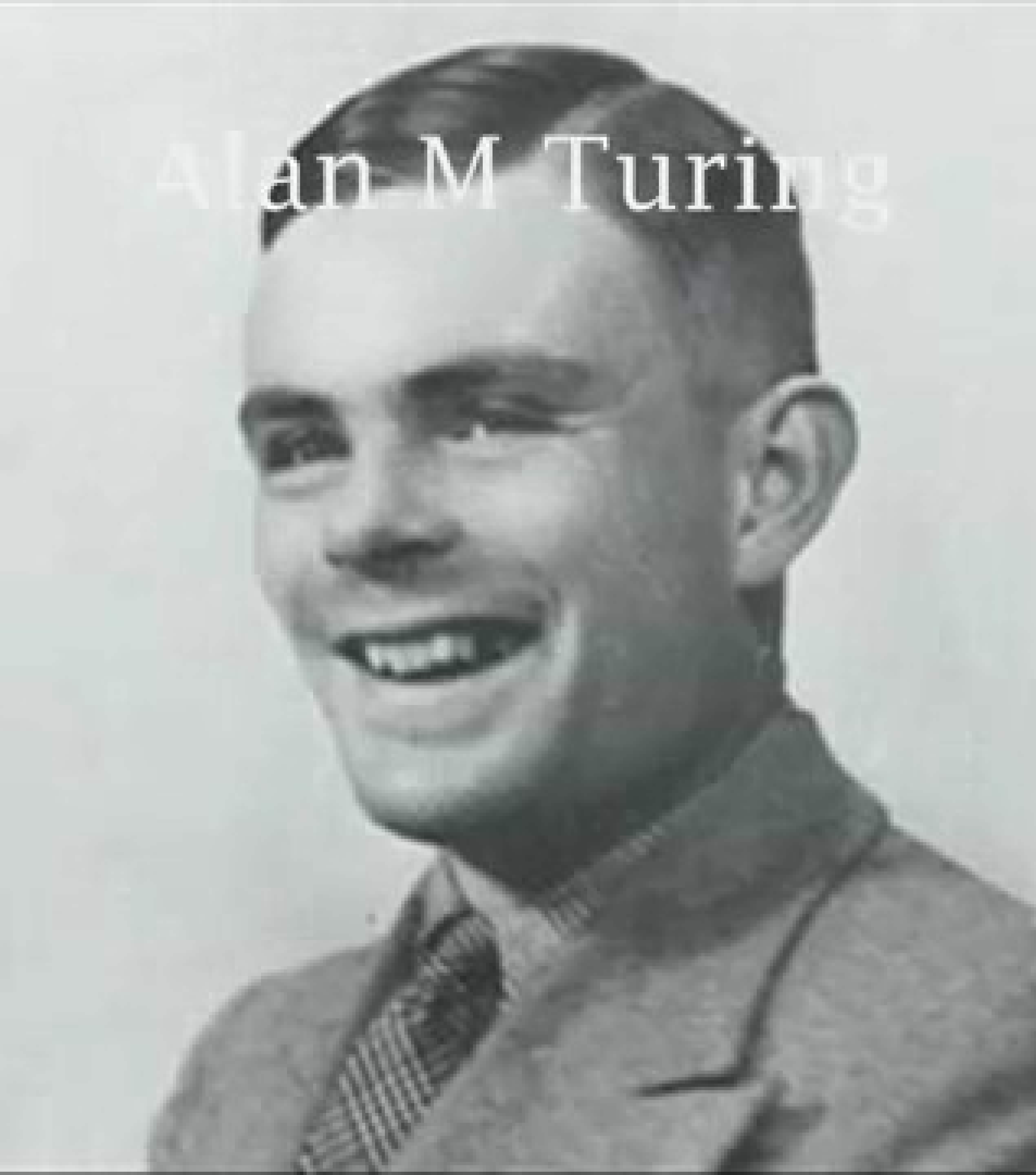
London 1912
Wilmslow 1954

- Mathematician
- Philosopher
- Father of Computer Science
- Cryptanalyst
- Visionary
- Homosexual
- Victim of prejudice

Turing: Biographical Data

- June 23rd 1912: Born in London (Paddington).
  His father was a successful public servant in India and his mother was a daughter of an important engineer in Madras. The children were raised by a retired colonel until the beginning of the war, after which their mother remained at home, in England.
- 1926: Boarding-school (Sherborne). Shy, introvert, clumsy, untidy, bad hand-writing, athletic.
- 1930: His first love, Chris Morcom, died of tuberculosis.
- 1931-1934: Scholarship for King’s College, Cambridge (Hardy, Eddington, Newman).
- 1935: Fellow at Cambridge.
- 1936-1938: PhD (scholarship) at Princeton. He met and associated with Church, Kleene, Gödel, Einstein, Von Neumann.
- 1939: Several discussions with Wittgenstein.
- 1939-1942: Bletchley Park: Decrypted the German Code (Enigma). In Bletchley Park he got engaged to Joan Clarke, but then separated. His contribution to the victorious outcome of the war was not publicly recognized, for security reasons.
- 1947: Meeting with Zuse.   1948: Design of Mark I.
- 1951: A brief relationship with a 19 year old man, who robbed him.
- 1952: Conviction. He was forced to take medication for his homosexuality.
- 1953: Restrictive measures, no security clearence. Vacation in Greece.
- June 7th 1954: Suicide by eating a poisoned apple (theater play “Breaking the Code”).
- 1966: Institution of ACM's Computer Science Award: Turing Award
- September 10th 2009: The British Government (Gordon Brown) publicly apologizes.

Alan M Turing




Turing

- At the age of 15 he compiled a summary of Einstein's Theory of Relativity.
- He discovered on his own and proved independently the Central Limit Theorem. He studied Probability Theory and Quantum-Mechanics.
- 1935: Fellow at Cambridge. He studied Integral Equations.
- 1935: He attended lectures of Max Newman, on the Foundations of Mathematics, culminating at the Incompleteness Theorem of Gödel.
- This is how he became interested in the Entscheidungsproblem of Hilbert.
  - His interest shifted from the list of rules of an algorithm to what we actually do when we execute them.
  - He proved that the actions we do could be limited to some extremely simple basic operations which, furthermore, could be executed mechanically (automatically).
  - He proved that by executing these basic operations no machine could determine whether a suggested conclusion results from given premises with the use of Frege rules.
  - Therefore, there is no algorithm for the Entscheidungsproblem.
  - An algorithmic solution to the Entscheidungsproblem would imply that every mathematical problem can be solved by using an algorithm.

Turing Machines (TM)

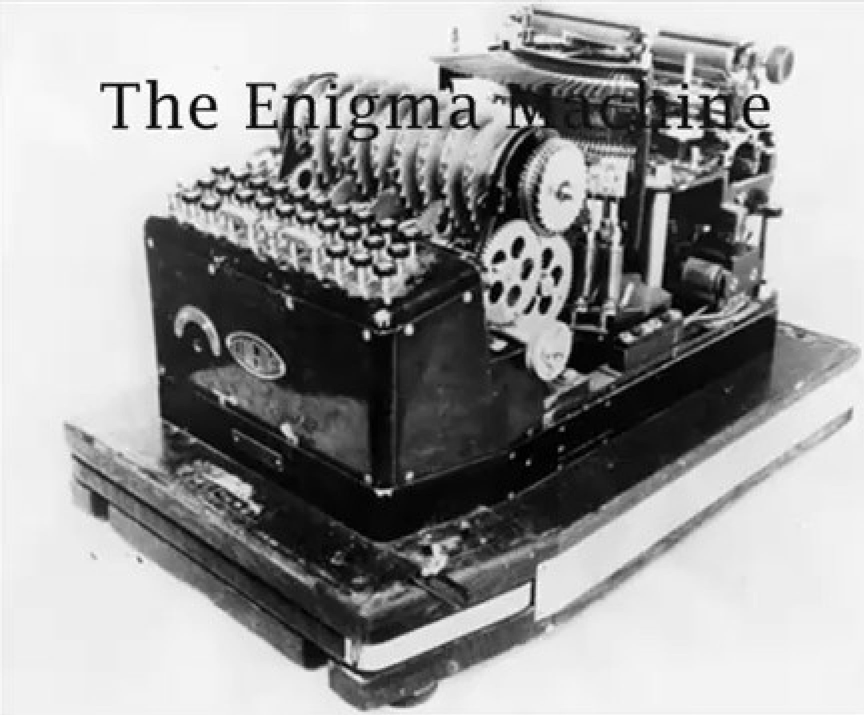
- He designed a simple machine which could compute anything that can be computed. Today, we call it a Turing Machine (TM).
- All computations are made on a tape, which is then scanned by a head that can read and write on every cell.
- The rules take into consideration the current symbol on the tape as well as the current state of the machine, and then decide which symbol will be written, what the next state will be and in which direction the head will move.
  - The machine, ultimately, was a series of quintuples (symbol read,previous state, symbol written, next state, movement).
  - The machine accepts its input if it stops.
- On the basis of Turing's analysis of the meaning of computation it is possible to conclude that anything that can be computed by any algorithmic procedure can also be computed by a TM. (Church-Turing Thesis)
- He constructed a universal TM which, all by itself, could do everything that any other TM could do, i.e., a model for a general purpose computer.
- He proved that the Halting problem is unsolvable.

Unsolvable problems

- In 1936 he published a paper, which is easy to read even today, *“On Computable Numbers, with an Application to the Entscheidungsproblem.”*
- Turing encoded the quintuples of the TM with natural numbers and by applying diagonalization, he proved the unsolvability of the Entscheidungsproblem.
  He used the code number of a TM as an input for a Universal TM.
- He showed that there is no substantial difference between hardware, software and data.
- Alonzo Church had a similar result using lambda-calculus and general Recursion.
- Under Church’s supervision he completed his doctoral dissertation in Princeton, repeatedly including undecidable propositions as axioms in new systems (Relativized Recursion 1938).
- Hierarchy of unsolvable problems. TM with an Oracle.
- An algorithmic solution to the Entscheidungsproblem would imply that every mathematical problem can be solved by using an algorithm.

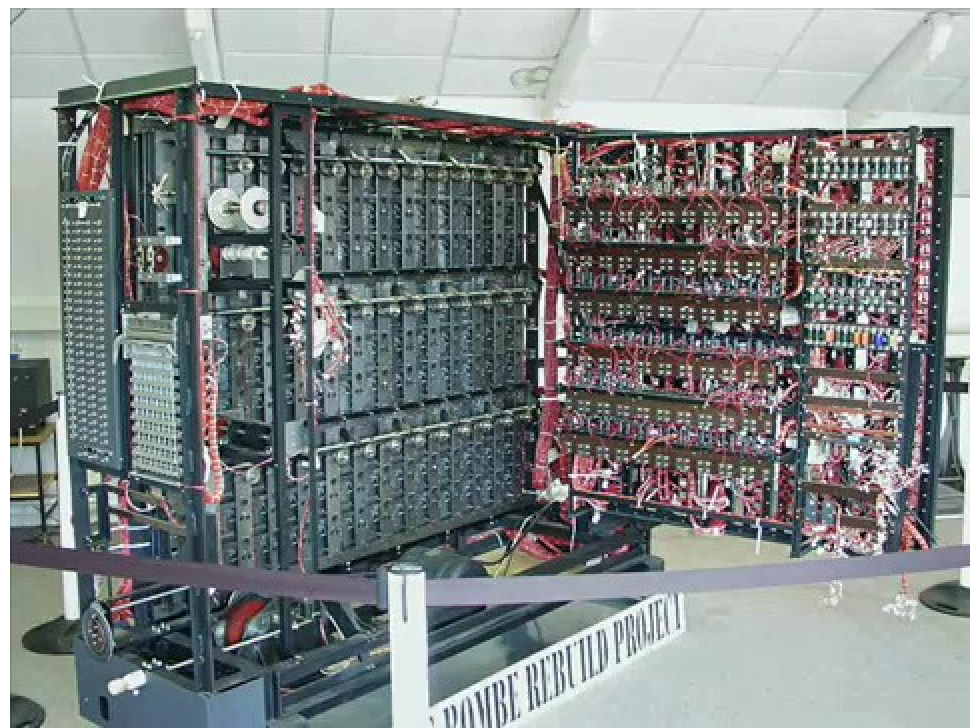
Turing & Enigma: Codebreaking-Cryptanalysis

- Enigma: a German invention for encrypted communication.
- 150.000.000.000.000.000.000 different combinations.
- Head of the British deciphering department (1940, Bletchley Park).
- Based on a book of codes found in a captured submarine and occasional carelessness of German encrypted messages, Turing constructed the “Bombe” machine, which could often solve messages sent by the German Enigma in less than 3 hours. The Bombe was based on existing cryptanalytic work by Polish mathematicians.
- With intelligent guessing and mathematical insight the time required for deciphering was reduced to 15 minutes.
- 1943-1945: Main consultant for British-American cryptanalysis.
- 1945: In collaboration with American allies, he designed Colossus (using vacuum tubes).



German encrypting machine.

The Bombe: Decrypting machine



There were 200 such machines at the end of the war

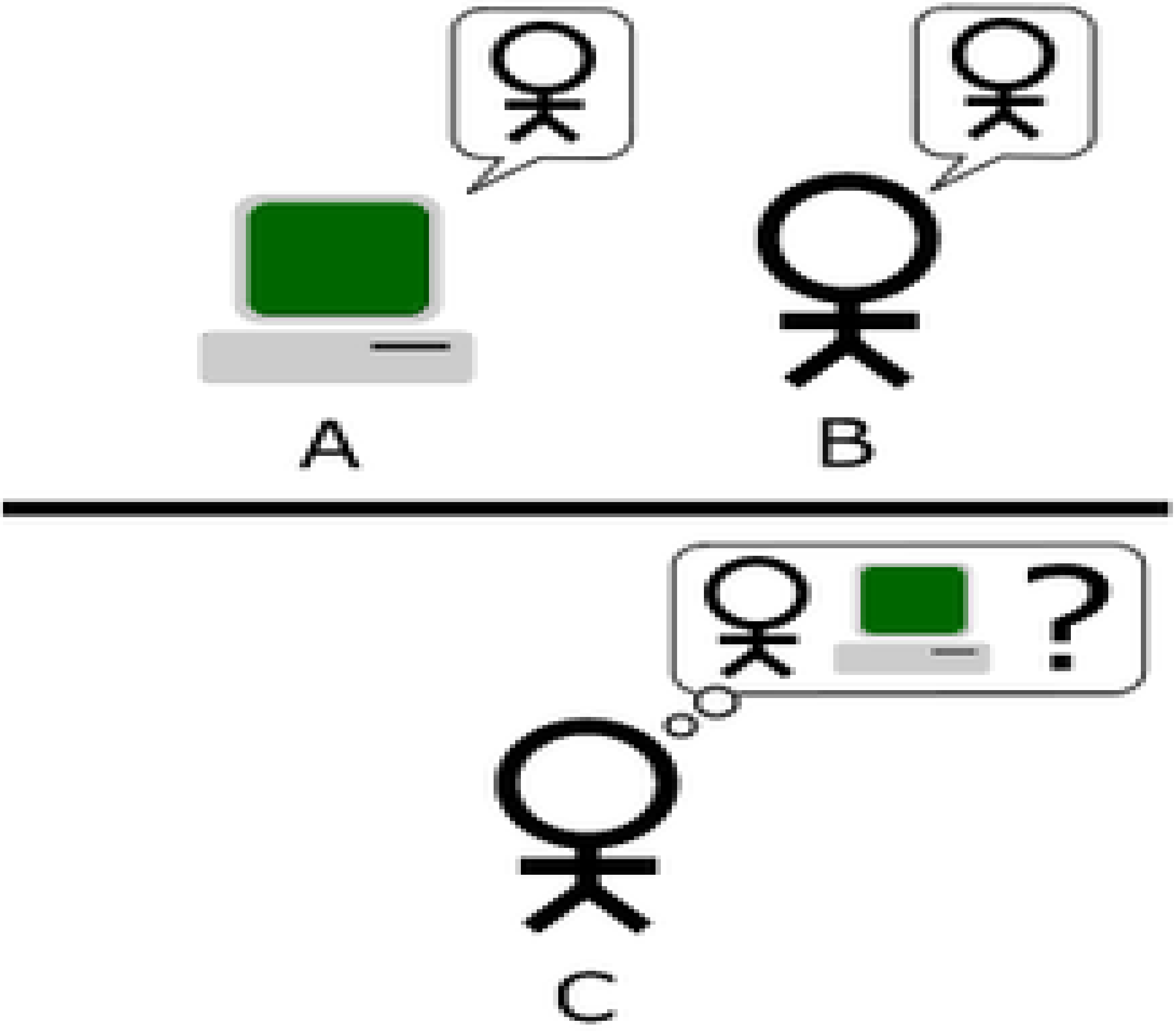
Alan Turing’s ACE

- 1945: He extended the specifications of EDVAC, transcending the limits of mathematical computations, by including problems like chess and puzzles. It was not implemented for lack of finances.
  - It reflected the <u>different</u> views of von Neumann and Turing.
- Minimalist regarding hardware, emphasizing software. Logical basic operations.
  - “[It] is… very contrary to the line of development here, and much more in the American tradition of solving one’s difficulties by means of much equipment rather than by thought… Furthermore, certain operations which we regard as more fundamental than addition and multiplication have been omitted.”

1946: Computer and software design.
1947: Programming, Neural Networks, Artificial Intelligence.
1948: University of Manchester.

1950. One step further: Turing's Test

- "…if a machine is expected to be infallible, it cannot also be intelligent…".
- Beginnings of Artificial Intelligence.
- Not "can machines think?"
  but "can machines do what humans do?"
- From electronic computer to intelligent brain.
- Dualism: the brain is not purely a physical construct.
- Materialism: there is a purely physical explanation for the brain, therefore artificial intelligence is possible.
- "It would be fair to say that a computer is intelligent if it can convince (deceive) a human that it (i.e. the computer) is human."
- 1950: "Computing Machinery and Intelligence".
- Papers in Biology.


A
B
?
C

Video Life

We now return to the journey from syllogisms to computations.

Gottfried Leibniz: Vision-Program

- Lingua Universallis, Algebraization of Logic, a Formal Language and a Formal System for Mathematics, Symbolic Logic, Algorithmic Decision.
- Vision
  - Leibniz believed that a system of appropriately chosen symbols is useful and indeed indispensable for deductive thought. He was seeking for a universal artificial mathematical language in which every kind of knowledge could be expressed.
  - Also, for computational rules that could reveal every logical dependence and implication, including mathematical statements.
  - Finally, he was seeking for machines that could compute, thus freeing the mind for more creative thoughts.
- Program
  - A symbolic language for every kind of knowledge (thus, specifically for mathematics).
  - A theory (a calculus) for every kind of knowledge (thus, specifically for mathematics).
  - A proof that this theory is consistent, i.e. it cannot deduce a contradiction (A and not A).

George Boole

- Algebra's strength emanates from the fact that the symbols that represent quantities and operations obey a small number of rules or laws.
- He strongly believed in the realization of Leibniz's dream.
- He wrote xy for the class of items belonging both to class x and to class y (intersection).
  - e.g. if x stands for “white things” and y for “sheep”, then xy stands for “white sheep”.
- He claimed that this resembles multiplication for numbers, with the exception that yy=y (cf. to A $\oplus$ A = A of Leibniz).

Boole: Algebra for Logic

- Two values 0 and 1.
- x+y denotes the class of all items belonging either to x or to y (union).
- x-y denotes the class of all items belonging to x but not to y.
- Some equations:
  - $x + (1-x) = 1$
  - $x(1-x) = 0$

Cantor: Diagonalization.

There are many different infinities.

Gottlob Frege

- He sought to construct a logical system that would entail all deductive argumentation in mathematical practice.
- He considered it important to invent his own special symbols for logical relations in order to avoid confusion with algebra and arithmetic.
  - $\neg$, $\wedge$, v , $\supset$, $\forall$, $\exists$.
  - ($\forall$x) (if x is a horse, then x is a mammal)
  - ($\exists$x) (x is a horse and x is pure-bred)
  - Everyone loves someone: ($\forall$x) ($\exists$y) L(x,y)
  - Everyone loves a lover: ($\forall$x) ($\forall$y) [($\exists$z) L(y,z) $\supset$ L(x,y)]
- Using sets, he defined cardinal and ordinal numbers.

Frege: Begriffsschrift

- Begriffsschrift (Concept Script) Mode of writing: Published in 1879, subtitled “A formula language, modeled upon that of arithmetic, for pure thought”.
  - The first example of a formal language with a strict syntax.
- Frege's logic has become the standard logic taught to undergraduate students in logic courses in mathematics, computer science and philosophy departments.
- Frege's logic was an enormous advance over Boole’s algebraic notation.

Frege: Bertrand Russel's letter (1902)

- "I agree with you on all the basics... there is just one point that I have encountered a difficulty".

  Frege added to his work:
  "there is nothing worse that can happen to a scientist than to have the foundation collapsed just as the work is finished. I have been placed in this position by a letter from Mr. Bertrand Russell."
  - $A = \{x \mid x \notin x\}$

    $A \in A \leftrightarrow A \notin A$ Contradiction

Frege never managed to recover from the blow.

Frege: Leibniz' dream

- Frege thought of his Begriffsschrift as embodying the universal language of logic that Leibniz had called for. However, Frege's rules provide no calculational procedures for determining whether some desired conclusion can be deduced from given premises in the logic of his Begriffsschrift.
- Because the Begriffsschrift did fully encapsulate the logic used in ordinary mathematics, it became possible for mathematical activity itself to be investigated by mathematical methods. As we will see later on, these investigations led to some remarkable and unexpected developments.

The Universal Computer

- The search for a computational method that could show whether a proposed conclusion in Frege's logic is valid was settled in 1936, with the proof that <u>such a general method does not exist</u>. This was a very unpleasant development for Leibniz's vision.
  However, during the procedure of proving this negative result, Alan Turing discovered something which would have made Leibniz very happy: that it was possible, theoretically, to construct <u>a universal machine which could, on its own, execute every possible computation</u>.

Hilbert: The 1900 Conference

- 23 problems that seemed difficult to solve using the existing methods.
  - "… every well-defined mathematical problem must have an exact solution…"
  - The list included: Cantor's Continuum Hypothesis, the Consistency of Arithmetic, Diophantine Analysis.
  - The set of problems he proposed has fascinated generations of mathematicians and determined developments in mathematics of the 20th century.
- The concept of existence:
  - "if it can be proved that properties of a concept can never lead to contradiction by applying a finite number of logical operations, then I say that mathematical existence of that concept has thus thereby been proven."
- On opposite camps with Poincaré.
  - Hilbert's circular reasoning: He used the very methods he wished to justify in the supposed proof that those methods cannot lead to contradiction.
  - Hilbert's answer: Metamathematics. Mathematics dealing with mathematical concepts like proof, consistency etc.
  - Tried to ridicule the formal proof method of theorems from axioms: the Chicago Machine in which you insert pigs and they come out as sausages and ham.

Hilbert’s Program

- Mathematics and logic should be developed together in a purely formal symbolic language. (Formalist school)
  - Metamathematics or Proof Theory.
  - The desired proof of consistency should be realized within metamathematics.
  - “I have proven that mathematicians will never run into contradiction using their usual methods, and I’ ve proven that using methods of which even you approve.”
- Completeness of First-Order Logic (i.e. the quest for an Axiomatic Formal System in which all valid propositions can be proven).
- Entscheidungsproblem – Decidability Problem (i.e. search for a method by which, given a proposition, it can be decided purely in a mechanic way algorithmically whether it is valid or not.)
- PA’s Completeness.
- Hilbert and Bernays: Grundlagen der Mathematik.

Kurt Gödel

Austria-Hungary

Brno 1906

Princeton 1978

Gödel: Completeness

- Principia Mathematica (Russell).
- Tractatus Logicophilosophicus (Wittgenstein).
  - Logical Systems can not only formalize mathematical reasoning, they can also be subjects of study themselves with the use of mathematical methods.
  - Hilbert: Are the rules of logic complete?
  - Or are there statements that, although true, not provable by the assumptions only using the rules.
- Gödel proved completeness in his PhD Thesis.
  - He only used methods that were already known.

Gödel: Upset

- 1931: «On formally undecidable propositions of Principia Mathematica and Related Systems».
  - Even stronger systems are incapable of including the full spectrum of mathematical proof.
  - There are statements that can be expressed but not proven inside those mathematical systems.
  - The critical point in the proof:

    The property of a natural number to be the code (Gödel number) of a provable statement in PM can itself be expressed in PM.
  - He used Diagonalization (Cantor).

Gödel: Upset

- Constructed a statement U: "U is not provable in PM" If we assume that PM doesn't lie, then U is true but not provable:
  - U is true. If U were false, then whatever it says holds. Thus it is provable. Therefore it is true.
  - U is not provable. Because it is true, whatever it says holds.
  - The negation of U is not provable in PM. Because U is true, ¬U is false and thus not provable in PM.
- If PM is consistent, then U holds.
  - Only the additional assumption that PM is consistent prevents proving U inside PM.
  - PM's consistency cannot be proven inside PM.
- Hilbert's program is dead.
- Gödel proved that the same metamathematical notions can be embedded in the language of formal logic itself.
- Undecidable Statements.

Turing: Beyond Leibniz's Dream

- Turing: «I believe that computing machines will eventually stimulate interest in symbolic logic… the language with which we communicate with these machines… is a kind of symbolic logic.»
- Computation with numbers is a form of reasoning and a large part of human reasoning can be considered as a kind of computation.
- In 1950 Turing foresaw that, until the end of the century, computers would be able to interact with humans, so that humans cannot distinguish if they are conversing with a human or a computer (computing machines and intelligence).
- Computers have no knowledge, they don't even know what it is they are computing.

John Von Neumann

Austria-Hungary

Budapest 1903

Washington 1954

Admitted that the main idea of modern computers stems from Turing's paper.

Von Neumann: the first digital computer

- The first who realized the importance of Gödel's incompleteness theorem.
  - "…I was able to prove that consistency of mathematics is not provable."
  - But Gödel had already reached that result.
- After Gödel's announcement he swore never to be involved in logic again.
- He joined the group that built the ENIAC in H.M. Moore School in Philadelphia (Eckert, Mauchly, Goldstine).
  - ENIAC had 18000 vacuum tubes modeled on differential analyzers. But it wasn't an analog machine.
- 1945: "First Draft on a Report on the EDVAC"
  - ENIAC's successor, implementation of the Universal Turing Machine.
  - Binary representation of numbers
  - Von Neumann Architecture: CPU (ALU), Bus, Memory.
  - Von Neumann appropriated the whole of the research effort.
- "It is easy to see by formal-logical methods that there exist codes that are *in abstracto* adequate to control and cause the execution of any sequence of operations which are individually available in the machine and which are, in their entirety, conceivable by the problem planner."
- This computer had memory that could save not only data but also programs.

Inventors of computing machines

- Pascal
- Leibniz
- Babbage
- Zuse
- Aiken
  - “if eventually it turns out that the basic principles of a machine designed for numerical solutions of differential equations would coincide with a principle of a machine whose purpose is the issuance of bills for a departement store, I would considered this the most fascinating coincidence that I have ever encountered.”
  - But in 1956 commercial computers that could do both were already available.

Turing: Next steps

- Cambridge 1949: construction of EDSAC by Wilkes modeled after ACE.
- Manchester 1949: construction of Mark I by Williams modeled after ENIAC using as memory prepared cathode ray tubes.
  - Later Turing was directing this project without being able to enforce his views.
  - However, he used this machine based solely on binary 0 and 1.
- Turing had a different view (than Von Neumann) about programming and translation.
  - Von Neumann: a simple bureaucratic task that had to be carried out manually without wasting time of a valuable scientific instrument.
  - Turing: “It must be fascinating. There is no real danger that it may turn boring because all automatic processes could be executed by the machine itself.”

Epilogue

- “We have followed the lives of a group of brilliant innovators spanning three centuries. Each of them in one way or another was concerned with the nature of human reason.Their individual contributions added up to the intellectual matrix, out of which emerged the all-purpose digital computer.”
- Except for Turing, none of them had any idea that his work might have such a tremendous application to our contemporary world.

Epilogue

- Leibniz saw far, but not that far.
- Boole could hardly have imagined that his algebra of logic would be used to design complex electric circuits.
- Frege would have been amazed to find equivalents of his logical rules incorporated into computer programs for carrying out deductions.
- Cantor certainly never anticipated the ramifications of his diagonal method.
- Hilbert's program to secure the foundations of mathematics was pointed in a very different direction.
- And Gödel, living his life of the mind, hardly thought of application to mechanical devices.

However:

Turing's great vision has already been realized.

Ref: Martin Davis