



# What is the Nature of Logic?

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# Outline of this Talk

1. What is the nature of logic? Some history
2. Maddy takes on a modern view
  - Logic is what holds on KF model possibilities
  - Natural deduction system
  - Pelham comments
3. Maddy argues logic is in the world
  - Pelham comments
4. Maddy argues logic is a cognitive mechanism
  - Pelham comments

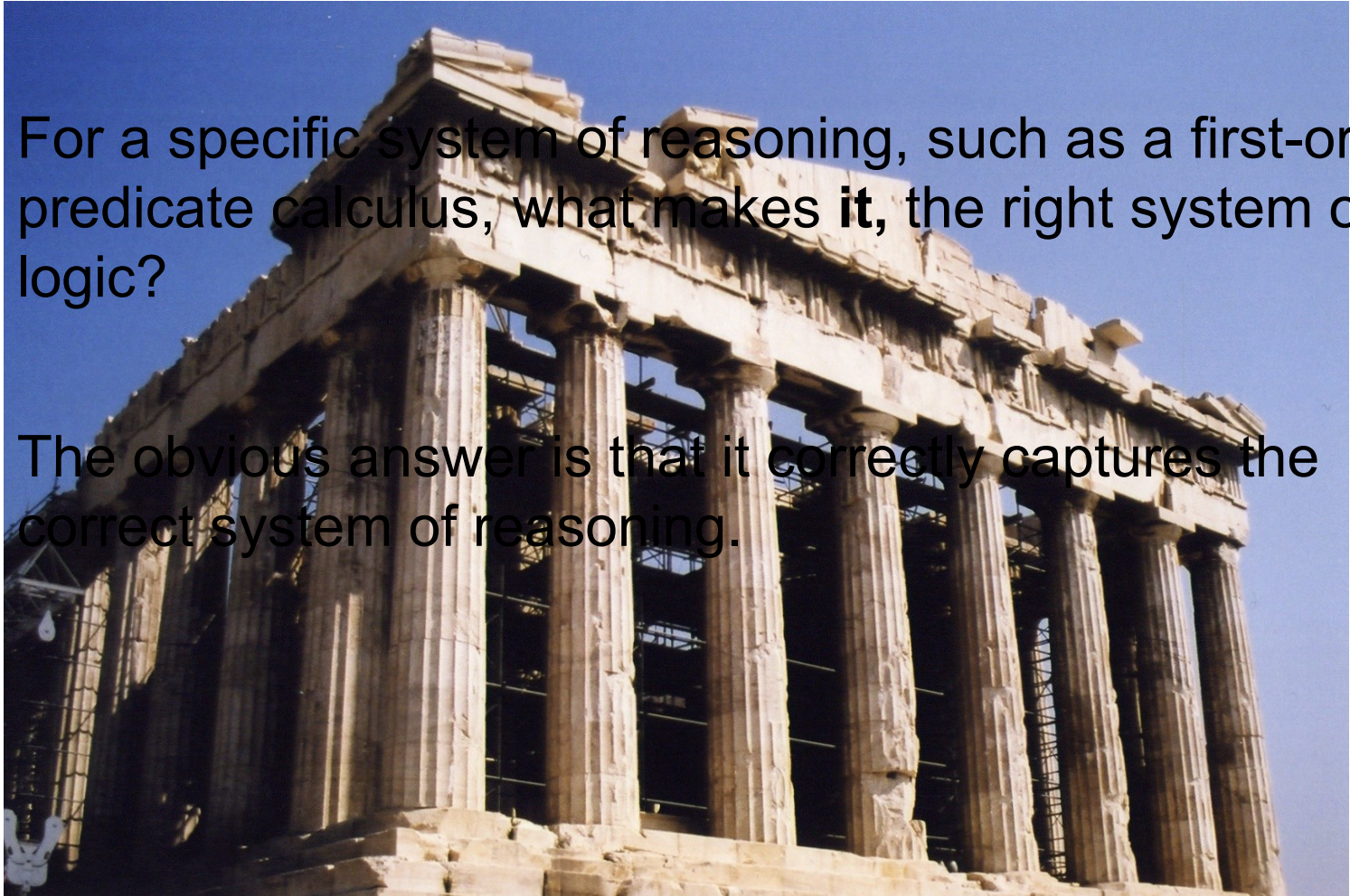
# Logic

- Logic is the study of correct reasoning.
  - What reasoning is correct?
    - There is some reasoning that people just DO. And that is the description of reasoning. That *may* be what psychologists study.
    - There is reasoning that people *should* do. This makes logic a rule-governed phenomenon. Logicians have been sensitive to study THIS.
  - What is logic,(as a systematic study)?
    - Aristotelian syllogistic reasoning
    - Frege-Russell predicate logic and its descendents

# What foundation can logic have?

For a specific system of reasoning, such as a first-order predicate calculus, what makes **it**, the right system of logic?

The obvious answer is that it correctly captures the correct system of reasoning.



# What can justify a certain system of logic as correct?

To ask this question is to ask, what makes, THIS system of logic (classical first order logic, intuitionistic logic, etc.) the system that justifies these steps of reasoning as the right ones?

# Some historical and philosophical answers:

- a. Nothing.
- b. Induction.
- c. The nature of space and time.
- d. Realism.
- e. Language

# Nothing can justify logic

- Because it is simply too basic, it is what grounds all our thinking.
- This is a good answer only when you are convinced that it is obvious that the logic you are working with is correct.

# Induction

- Hume and J.S. Mill thought that we only arrived at the conclusion that a logical rule was right because we had seen it work so many times and never seen that it does not work.
- Frege ridiculed this type of response as “pebble arithmetic”.



# Space and time

- Kant thought that since our knowledge of the world was controlled by our perceptions, and
- Since our perceptions naturally inclined us to think in terms of events happening in space and time,
- Logical truths emerge as a priori, and what is a priori is before our experience. Logical truths are analytic and a priori, mathematical truths are synthetic and a priori.

# Realism

- Frege was a realist about logical form. That a conclusion followed from some premises logically was a fact of the world that pertained to specific concepts.
- Russell was a realist about propositions. Logical truths are true in virtue of propositions being structured in the way that they are.

# Language

- Wittgenstein originally argued in *TLP* that logical rules and tautologies are true in virtue of the structure of representation.
- Logical positivists took this idea to mean that logical truths were true in virtue of language.
- Carnap said that to adopt a logic was to adopt a language, and there could be different logics/languages for different purposes.

# More language

- Quine explained that all of our language is a theoretical construction that allows us to make sense of the world, and logical truths are the most fundamental of these. They are theoretically changeable, but in practice, we cannot change them, or else we cannot make sense of anything.
- We further counselled that we should see our knowledge as arriving, 'naturally' that is, from our sensation and our activity in the world.

# Language & Analytic philosophy

- Quine's idea of language as fundamental to our acceptance of logic has remained in the background of philosophy.

But all these views really accept either Aristotlean or classical first order logic as “correct”.

None takes advantage of the richness of mathematical work in logic, there are distinct logical “systems” possible.

# Penelope Maddy as a “naturalist”

- Maddy’s 2007 book, *Second Philosophy: a naturalistic method* takes a different approach to justifying logic that is an attempt to justify logic as we would justify other branches of natural science.
- ‘Second philosophy’ is a method which rejects ‘first philosophy’, the search for an ultimate ground for knowledge. It adopts the view that each branch of science is open for revision while holding the other aspects constant.

## Maddy's philosophy of logic (sketch)

“(1) logic is true of the world because of its underlying structural features, and (2) human beings believe logical truths because their most primitive cognitive mechanisms allow them to detect and represent the aforementioned features of the world.” (Maddy, *Second Philosophy*, p. 226)

# Logic is based on KF-models

KF models analyze propositions into **object /n-ary relation form**.

- Maddy: “Speaking in complete abstraction, a KF world would consist of various objects,  $a, b, c, \dots$ , which enjoy various properties  $P, Q$ , and stand in various relations with various numbers of arguments,  $R, S, \dots$ ” (Maddy, p. 228)



# Logic as based on KF-models

- KF models contain the **possibility of universal statements**.
- Complex states are dependent on the simple states, and this includes (some!) truth functions and first order quantification.

# Logic as based on KF-models

- KF models have properties that may be related by causal relations.  $(\forall x)(Cx \rightarrow Ex)$
- Maddy calls these “**ground-consequent**” **relations**.
- Maddy, “some of the various states of our KF world may be interconnected: for example, it might be that every  $a$  for which  $Rac$  is also an  $a$  for which  $Pa$ . This might be, so to speak, an accidental connection, but if  $a$ 's bearing  $R$  to  $c$  is the ground of its having  $P$ , then this is a ground-consequent dependency.” (Maddy, p. 228)

# Logic is based on KF-models

- KF models permit **indeterminacy** in the world.
- Maddy: “For any property P, the domain of a KF world will divide into those objects that have P, and those which do not have P, and those for which P is indeterminate; and perhaps even these boundaries between these groupings are somewhat fuzzy.” (Maddy, p. 229)

# Different types of indeterminacy

- Vague predicates have indeterminate cases:
  - ‘x is tall’
  - ‘x is heavy’
  - ‘x is a heap’
- Predicates admit of category mistakes:
  - ‘x dreams’ does not apply to clouds
  - ‘x is red’ does not apply to sets
  - ‘x smells’ does not apply to colors

# “Rudimentary Logic”

- is the natural deduction system that Maddy adopts in view of the fact that **its** rules
- are valid on all KF models.

**Maddy’s “Rudimentary Logic”** contains:

Modus Ponens, disjunctive syllogism, conjunction intro and elimination, and double negation rules.

# Kleene Connectives for KF models

$\sim P$

P	$\sim P$
T	F
-	-
F	T

P&Q

&	T	-	F
T	T	-	F
-	-	-	F
F	F	F	F

PvQ

v	T	-	F
T	T	T	T
-	T	-	-
F	T	-	F

# Natural Deduction Rules valid in the class of all KF models

Disjunctive Syllogism	$\{P \vee Q, \sim Q\} \models P$	✓ = valid
Addition	$P \models P \vee Q$	✓
Conjunction Intro	$P, Q \models P \& Q$	✓
Conjunction Elimin	$P \& Q \models P$	✓
Modus Ponens	$P \rightarrow Q, P \models Q$	✓
Modus Tollens	$P \rightarrow Q, \sim Q \models \sim P$	✗ = invalid
Reductio ad Absurdum	$[P \models Q, \sim Q] \models \sim P$	✗
Tautologies	eg. $P \vee \sim P$	✗

## Maddy's KF models are innovative philosophically because they

- keep the traditional semantic framework of models grounding validity. But,
- they alter the models to adapt to indeterminacy **in the world**. (I think not all indeterminacy is explained by our lack of knowledge.)
- use indeterminacy to **explain** a deduction theorem free setting.
- Avoid commitment to reductio ad absurdum and modus tollens.



## Unfortunately, ...

- At the end of her section on the philosophy of logic, Maddy repudiates rudimentary logic in favour of classical logic!!
- On the grounds that we can **idealize away from** vagueness in some cases!!
- I would like to see work on models of KF types of structures to investigate how they would work to model causation... (open question)

# Model theory is adapted here!

- Maddy states,
  - “Rudimentary logic **is true of the world** insofar as it is a KF-world, and in many but not all respects it is.”
- Inference rules are true!
- This can only mean that a rule is true iff the subject matter that the rule applies to is correctly structured as a KF model is structured.
- This is a fundamental break from Tarski !

# The actual world is a KF-world (1)

- The actual world consists of objects. Obviously!!
- The category of objects is crucial to science, not what objects are held basic.
- But the two-slit experiment seems to suggest that objects are dispensable in science.

# The world is a KF world (2)

- The world contains
  - Properties and relations
  - Causation
    - This is easy to say, but not so easy to explain fully..
  - Indeterminacy
    - There is more work to do to explain how aspects of the world can be indeterminate.

# Revise Maddy

- We should take these considerations to show that different sorts of models apply in different situations.
- Actual human reasoning that we think of as necessary is what is common to all of the reasonably ordinary situations we see. Rudimentary logic is a first attempt at finding this.
- Other models are more rarified.
- The non-Tarskian use of model theory, and this conference, supports that view.

# Human Cognition

- The second aspect of Maddy's philosophy of logic is (2') "human beings believe the simple truths of rudimentary logic because their most primitive cognitive mechanisms allow them to detect and represent KF-structures."
- This is a notion of logic that would apply to all human beings! And it is still normative. It explains how we can "get it wrong."

# Argument for KF in human cognition

(1)

- Objects are central to human understanding of the world.
- This happens before infants have any language ability.

# Argument for infant recognition of

- Properties
- Causation

Is present although weaker than object research.

There is no discussion of how adults or infants deal with indeterminacy in reasoning.



# More data is needed!

- On the psychology of reasoning front in particular.
- Vanessa Lehan has found that a majority of reasoning experiments:
  - Presuppose classical logic in their design
  - The majority of experiments show that subjects fail with respect to contrapositive reasoning and paradoxes of material implication disproportionately to how they do with respect to other rules.